On the Møller Energy-Momentum Complex of the Melvin Magnetic Universe

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Abstract

We use the Møller energy-momentum complex to calculate the energy of the Melvin magnetic universe. The energy distribution depends on the magnetic field.

Keywords: Møller energy-momentum complex, Melvin magnetic universe.

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Introduction

The subject of energy-momentum localization in general relativity continues to be an open one because there is no given yet a generally accepted expression for the energy-momentum density. Even they are coordinate dependent, various energy-momentum complexes give the same energy distribution for a given space-time.

Aguirregabiria, Chamorro and Virbhadra [1] obtained that the energy-momentum complexes of Einstein [2], Landau and Lifshitz [3], Papapetrou [4], and Weinberg [5] give the same result for the energy distribution for any

Kerr–Schild metric. Also, recently, Virbhadra investigated [6] if these definitions lead to the same result for the most general nonstatic spherically symmetric metric and found they disagree. Only the energy-momentum complex of Einstein gives the same expression for the energy when the calculations are performed in the Kerr–Schild Cartesian and Schwarzschild Cartesian coordinates. The Møller energy-momentum complex allows to compute the energy in any coordinate system.

Some results recently obtained [7]-[10] sustain that the Møller energy-momentum complex is a good tool for obtaining the energy distribution in a given space-time. Also, in his recent paper, Lessner [11] gave his opinion that the Møller definition is a powerful concept of energy and momentum in general relativity. Very important is the Cooperstock [12] hypothesis which states that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields. Also, Chang, Nester and Chen [13] showed that the energy-momentum complexes are actually quasilocal and legitimate expression for the energy-momentum.

In this paper we calculate the energy distribution of the Melvin magnetic universe in the Møller prescription. We use geometrized units ($G = 1, c = 1$) and follow the convention that Latin indices run from 0 to 3.

Energy in the Møller Prescription

The Melvin magnetic universe [14], [15] is described by the electrovac solution to the Einstein–Maxwell equations and consists in a collection of parallel magnetic lines of forces in equilibrium under their mutual gravitational attraction. The Einstein–Maxwell equations are

$$R_i^{\ k} - \frac{1}{2}\, g_i^{\ k}\, R = 8\,\pi\, T_i^{\ k}, \tag{1}$$

$$\frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, F^{ik}\right)_{,k} = 4\,\pi\, J^i, \tag{2}$$

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0. \tag{3}$$

The energy-momentum tensor of the electromagnetic field is given by

$$T_i^{\ k} = \frac{1}{4\,\pi}\left[-F_{im}\, F^{km} + \frac{1}{4}\, g_i^{\ k}\, F_{mn}\, F^{mn}\right]. \tag{4}$$

The electrovac solution corresponds to $J^i = 0$ and is given by the metric

$$ds^2 = L^2\,(dt^2 - dr^2 - r^2\,d\theta^2) - L^{-2}\;r^2\,\sin^2\theta\;d\varphi^2, \tag{5}$$

where

$$L = 1 + \frac{1}{4}\,B_0^2\,r^2\,\sin^2\theta. \tag{6}$$

The Cartan components of the magnetic field are

$$\begin{aligned} H_r &= L^{-2}\;B_0\,\cos\theta, \\ H_\theta &= -L^{-2}\;B_0\,\sin\theta. \end{aligned} \tag{7}$$

B_0 is the magnetic field parameter and is a constant in the solution given by (5) and (6).

The energy-momentum tensor has the non-vanishing components

$$\begin{aligned} T_1^{\;1} &= -T_2^{\;2} = \frac{B_0^2\,(1 - 2\,\sin^2\theta)}{8\,\pi\,L^4}, \\ T_0^{\;0} &= -T_3^{\;3} = \frac{B_0^2}{8\,\pi\,L^4}, \\ T_2^{\;1} &= -T_1^{\;2} = \frac{2\,B_0^2\,\sin\theta\,\cos\theta}{8\,\pi\,L^4}. \end{aligned} \tag{8}$$

The Møller energy-momentum complex $M_i^{\;k}$ [16] is given by

$$M_i^{\;k} = \frac{1}{8\,\pi}\,\chi_i^{\;kl}{}_{,l}, \tag{9}$$

where

$$\chi_i^{\;kl} = -\chi_i^{\;lk} = \sqrt{-g}\,\left(\frac{\partial g_{in}}{\partial x^m} - \frac{\partial g_{im}}{\partial x^n}\right)\,g^{km}\,g^{nl}. \tag{10}$$

Also, $M_i^{\;k}$ satisfies the local conservations laws

$$\frac{\partial M_i^{\;k}}{\partial x^k} = 0. \tag{11}$$

$M_0^{\;0}$ is the energy density and $M_\alpha^{\;0}$ are the momentum density components.

The energy and momentum are given by

$$E = \iiint {M_0}^0 dx^1 dx^2 dx^3 = \frac{1}{8\,\pi} \iiint \frac{\partial {\chi_0}^{0l}}{\partial x^l}\, dx^1\, dx^2\, dx^3. \tag{12}$$

For the Melvin magnetic universe we obtain

$$\begin{aligned} {\chi_0}^{01} &= \frac{B_0^2\, r^3\, \sin^3\theta}{(1 + 1/4\, B_0^2\, r^2\, \sin^2\theta)}, \\ {\chi_0}^{02} &= \frac{B_0^2\, r^2\, \cos\theta\, \sin^2\theta}{(1 + 1/4\, B_0^2\, r^2\, \sin^2\theta)}. \end{aligned} \tag{13}$$

After some calculations, applying the Gauss theorem and plugging (13) into (12) we obtain the energy distribution

$$E(r) = \frac{1}{3}\, B_0^2\, r^3 - \frac{1}{15}\, B_0^4\, r^5 + \frac{1}{70}\, B_0^6\, r^7. \tag{14}$$

Put the G and c at their places we get

$$E(r) = \frac{1}{3}\, B_0^2\, r^3 - \frac{1}{15}\, \frac{G}{c^4}\, B_0^4\, r^5 + \frac{1}{70}\, \frac{G^2}{c^8}\, B_0^6\, r^7. \tag{15}$$

The first term represents twice of the classical value of energy [17] obtained in the Landau and Lifshitz and Papapetrou prescriptions. The other terms are due to the relativistic correction.

Discussion

Many results recently obtained sustain the viewpoint of Bondi [18]. He gave his opinion that a nonlocalizable form of energy is not admissible in relativity.

We obtain the energy distribution of the Melvin magnetic universe using the energy-momentum complex of Møller. The energy depends on the magnetic field. The result is different as that obtained by Xulu [17] using the energy-momentum complexes of Landau and Lifshitz and those of Papapetrou. The first term represents twice of the classical value of energy [17] obtained in the Landau and Lifshitz and Papapetrou prescriptions. The third term, that is due to the relativistic correction, is twice of value of energy obtained in [17]. Also, the Møller energy-momentum complex does not need to carry out calculations in any particular coordinates.

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